Optimal fast single-pulse readout of qubits

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The computer simulations of the process of single-pulse readout from a two state quantum system, describing the phase qubit, is performed in the frame of one-dimensional Schroedinger equation. It has been demonstrated that the readout error can be minimized by choosing the optimal pulse duration and the depth of a potential well, leading to the fidelity of 0.94 for 2 ns sinusoidal pulses.

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In the past decade considerable progress has been achieved in the development of various circuits for quantum computation.[1,2] Different sources of decoherence is, however, the main factor limiting practical utilization of the complex networks of quantum bits.[3–10] Recently, it has been demonstrated that the coherent Rabi oscillations remain nearly unaffected by thermal fluctuations up to temperatures of 1 K (Ref. 11) (i.e., until the energy of thermal fluctuations kT becomes comparable with the energy-level spacing $\hbar\omega$ of the qubit), so without degrading the already achieved coherence times, phase qubits can be operated at temperatures much higher than those reported so far. This may signal that relatively large readout errors of practical devices[4] can be attributed to nonoptimal readout of the qubits rather than quantum and thermal fluctuations. To speed up the readout, the fast single-pulse readout (FSPR) technique has been realized and tested.[3–8] An example of a shallow potential well with two energy levels $|0\rangle$ and $|1\rangle$ is presented in the inset of Fig. 1. The basic idea of the FSPR is to apply the readout pulse in such a way, that, while it is on, the system will tunnel from state $|1\rangle$ through the barrier with a probability close to unity, whereas from the state $|0\rangle$ it will tunnel with a probability close to zero. The effect of the shape and duration of the pulse on different error probabilities has been studied in Refs. 9 and 10. In particular, the error to excite higher qubit states due to nonadiabaticity of the pulse was analyzed.[10] As has been understood,[8,10] the main source of error during the qubit readout is due to incomplete discrimination between the two quantum states, $|0\rangle$ and $|1\rangle$. It is claimed that the quality of tunneling discrimination largely depends on the measurement pulse amplitude, with the error decreasing for longer measurement pulses and at larger Γ_1/Γ_0 ratio (where Γ_0 and Γ_1 are tunneling rates from the states $|0\rangle$ and $|1\rangle$, respectively). However, at finite temperatures longer readout will obviously lead to larger readout errors, since thermal activation over the barrier becomes possible. The main goal of the Brief Report is to analyze the opportunity to use very fast readout of order 1 ns with acceptably high fidelities to make the readout time much smaller than the qubit coherence time.

Recently, in the classical systems subjected to noise and pulsed or periodic driving, such as Josephson junctions[12] and magnetic nanoparticles,[13] it has been demonstrated that at a fixed value of the driving amplitude there exists an optimal pulse duration, which minimizes the noise-induced errors. It is intriguing to understand, if the effect discovered for classical systems with noise,[12,13] would be realized in a purely quantum system, e.g., as the example of a qubit described in Refs. 6, 7, and 10. It is known, that the Fokker-Planck equation for an overdamped Brownian particle subjected to noise by simple change of variables and change to imaginary time, can be transferred to the Schroedinger equation for a quantum particle. However, until now the results of Ref. 12 were not generalized for a purely quantum system, governed by the Schroedinger equation. In the present Brief Report we demonstrate, that the readout error of a quantum two state system, subjected to a pulse driving, also has a minimum as function of the pulse duration, as it was previously observed for classical systems with noise.[12,13] The application of this effect for qubits allows to decrease the readout time, making it much smaller than the qubit coherence time, and, therefore, improving the overall fidelity of the system.

Let us consider the example of a flux-biased phase qubit,[6,7,10] which is described by the following potential $V(x,t)=E_J\{(x-\varphi(t))^2/2\ell-\cos x\}$, see Fig. 1, dashed curve. Here $E_J=I_C\hbar/2e$ is the Josephson energy, x is the Josephson phase, e is the electron charge, and $\hbar$ is the Planck constant. For the qubit we take the same parameters as in Refs. 6 and 10: the critical current $I_C=1.7$ μA, the inductance of the ring $L=0.72$ nH and the capacitance $C=700$ fF, thus $\ell=2eI_CL/\hbar=3.71$, $2e^2/\hbar C=0.6933\times10^9$ Hz, $E_J/\hbar=I_C/2e=5.31\times10^{12}$ Hz, so it is convenient to introduce the "inverse capacitance" $D=2e^2/\hbar C\times10^{-9}$ and express the time in nanoseconds. The dimensionless external magnetic flux $\varphi(t)$ consists of two components: the dc component a_0, whose adjustment changes the depth of the shallow well, and the driving readout pulse: $\varphi(t)=2\pi[a_0+Af(t)]$, where A is

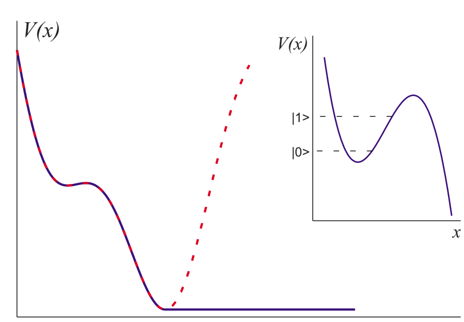

FIG. 1. (Color online) The profile of a bistable potential. Dashed curve—the original potential, solid curve—the potential with enlarged deep well to simulate the effect of damping. The inset: the enlargement of a shallow potential well.

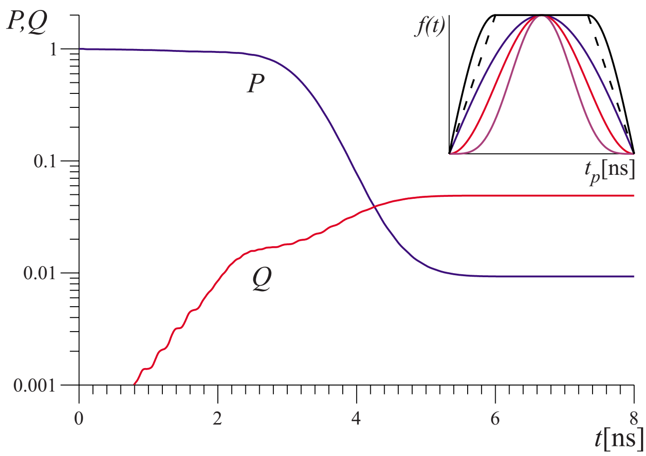

FIG. 2. (Color online) The evolution of probabilities $P(t)$ and $Q(t)$. Inset: the considered pulses, from top to bottom, sine-trapezoid pulse, trapezoidal pulse, $\sin(\pi t/t_p)$, $\sin^2(\pi t/t_p)$, and $\sin^4(\pi t/t_p)$.

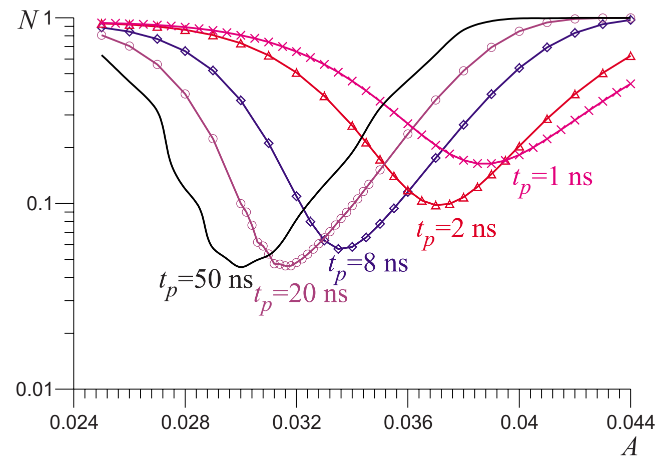

FIG. 3. (Color online) The readout error N versus the pulse amplitude A for the pulse $\sin(\pi t/t_p)$.

the pulse amplitude, and $f(t)$ is one of $\sin(\pi t/t_p)$, $\sin^2(\pi t/t_p)$, $\sin^4(\pi t/t_p)$, the trapezoid function, which linearly grows and drops for $t \geq t_p/4$ and $t \leq 3t_p/4$, and the sine-trapezoid function, which differs from the previous one by the sinusoidal walls, see the inset of Fig. 2. We note that t_p is defined as the full width of the pulse at zero level, rather than the full width at half maximum.[4] The shift of a potential barrier $a_0=0.81$ is chosen such as to allow the six levels to be present in a shallow potential well (see Ref. 10). The pulse with the amplitude $A \approx 0.035$ leads to lowering of the potential barrier so that only two levels will remain. Let us study the readout error N, which is the sum of two probabilities, P_{10} not to tunnel during the pulse action from the state $|1\rangle$, and P_{01} to tunnel from the state $|0\rangle$ (i.e., $N=P_{10}+P_{01}$, while the fidelity $F=1-N$). The investigation is performed via computer simulation of the Schroedinger equation and is focused on the readout error N versus the pulse amplitude, duration and shape, as well as the depth of a shallow potential well.

The Schroedinger equation for the wave function $\Psi(x,t)$ has the following form:

$$i\frac{\partial\Psi(x,t)}{\partial t} = -\frac{2e^2}{\hbar C}\frac{\partial^2\Psi(x,t)}{\partial x^2} + \frac{V(x,t)}{\hbar}\Psi(x,t). \tag{1}$$

It is assumed that the boundary conditions $\Psi(c,t)=\Psi(d,t)=0$ at the points c and d are taken far away from the shallow potential well and do not affect the tunneling process. To prevent the repopulation error,[10] which arises due to the absence of damping in our model, let us introduce effective damping in the following manner. Since we are interested in the process of tunneling from the shallow potential well only, we assume that at the bottom of the deep potential well the potential does not grow up, but spreads to the right really far away, see Fig. 1, solid curve. In particular, we have taken $c=-3$, $d=797$, while the left minimum is located at $x_1 \approx 1.4$, and the right one at $x_2 \approx 6$. Numerical solution of Eq. (1) has been performed on the basis of implicit finite-difference Crank-Nicholson scheme. The values of discretization steps are $\Delta x=0.01$, $\Delta t=10^{-4}$ ns, further decrease in the steps did not change the results.

In Fig. 2 the evolution of the probabilities $P(t)$ not to tunnel from the state $|1\rangle$ and $Q(t)$ to tunnel from the state $|0\rangle$ are presented. In both cases we choose the initial condition to correspond to either $|1\rangle$ or $|0\rangle$ stationary state and calculate the survival probabilities $P_1(t)$ and $P_0(t)$ in the shallow potential well. The probability evolutions $P(t)=P_1(t)$ and $Q(t)=1-P_0(t)$ are presented in Fig. 2 for the case of a sinusoidal pulse with the driving amplitude $A=0.034$ and pulse duration $t_p=8$ ns. One can see that at the end of the pulse both probabilities are significantly smaller than unity, and our task is to find the optimal parameters of the pulse to minimize the readout error $N=P_{10}+P_{01}=P(t_p)+Q(t_p)$.

The readout error N versus the pulse amplitude A for a sinusoidal pulse is presented in Fig. 3 for different values of the pulse width t_p. It is seen that N is very sensitive to the pulse amplitude, especially for large pulse durations, and that the minimal value of N decreases with an increase in t_p. A similar figure for one pulse width is presented in Ref. 8. However, from Fig. 3 one can see that for the pulse durations $t_p \geq 8$ ns the minimal value of N is almost the same, while below $t_p \leq 2$ ns N increases significantly.

The readout error N versus the pulse width t_p for the sinusoidal pulse is presented in Fig. 4 for different values of the pulse amplitude A. One can see that N has a strongly pronounced minimum, and with an increase in A this minimum shifts to smaller tunneling times, thus speeding up the readout, but the minimal value of the error N increases, so one should find a compromise between the error and the speed. However, in the range of small amplitudes N increases insignificantly (compare the curves for $A=0.03$, $A=0.032$, and $A=0.034$), and for the parameters in question the amplitude $A=0.034$ can be chosen as a compromise value, leading to fast readout with a high fidelity, $F=0.94$. The results presented in Figs. 3 and 4, on one hand, demonstrate a similar dependence of the readout error on pulse width, as for classical systems.[12,13] On the other hand, the dependence on the driving amplitude also has a minimum, which outlines the quantum nature of the described phenomena, while for a classical system the error monotonously de-

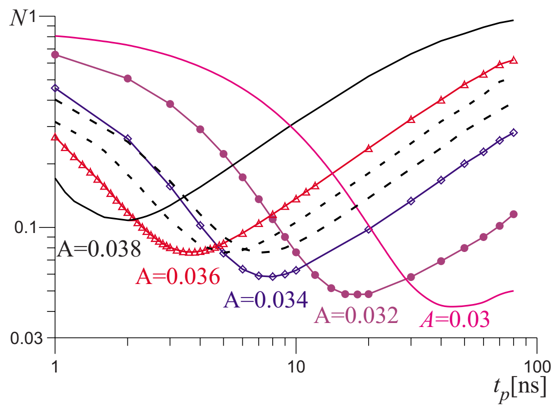

FIG. 4. (Color online) The readout error N versus the pulse width t_p for the pulse $\sin(\pi t/t_p)$. The dashed curves correspond to $\sin^2(\pi t/t_p)$, $\sin^4(\pi t/t_p)$ pulses with $A=0.036$ (from left to right).

creases with an increase in the pulse amplitude. Another important deviation from the classical system is the impossibility to use rectangular readout pulses (which in the classical case leads to minimal noise-induced errors). In the qubit a rectangular pulse leads to nonadiabaticity of the tunneling event, which results in lifting to higher eigenstates and considerable increase in the probability to tunnel from $|0\rangle$ state and thus to much larger values of N than for all other pulse shapes under study.

The readout error for different pulse shapes $\sin^2(\pi t/t_p)$ and $\sin^4(\pi t/t_p)$, $A=0.036$ is presented in Fig. 4 by dashed curves. As one can see, the minimal value of the error is pretty much the same, but the optimal readout time is significantly smaller for $\sin(\pi t/t_p)$ just due to the fact that the top of the pulse in this case is flatter. In accordance with these results the trapezoid pulse should lead to smaller N. This is indeed so, as one may see from Fig. 5, thick solid curve. However, the dependence of N vs t_p demonstrates oscillations, and it is not easy to properly choose the optimal

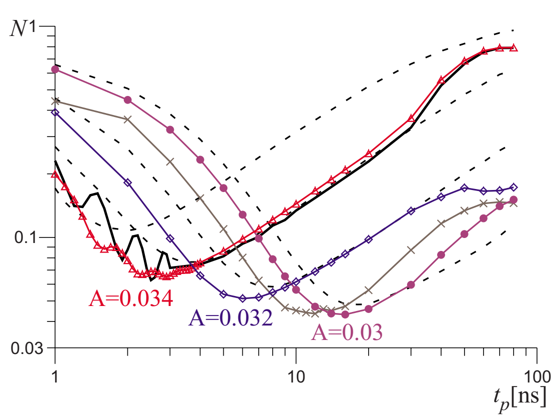

FIG. 5. (Color online) The readout error N versus the pulse width t_p for the trapezoid pulse with $A=0.034$ (thick solid curve) and sine-trapezoid pulses (solid curves with triangles, diamonds and circles for amplitudes $A=0.034;0.032;0.03$). The curves for the pulse shape $\sin(\pi t/t_p)$ from Fig. 4 with $A=0.038;0.036;0.034;0.032$ are given by dashed curves for comparison. Solid curve with crosses—long sine-trapezoid pulse for $A=0.03$.

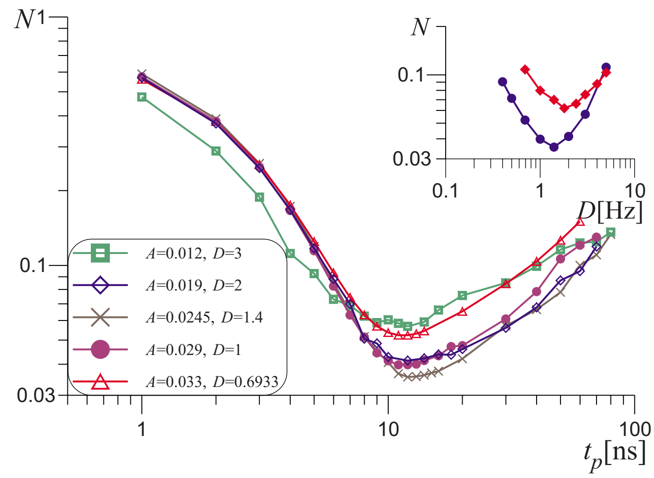

FIG. 6. (Color online) The readout error N versus the pulse width t_p for the pulse $\sin(\pi t/t_p)$ with different amplitudes and inverse capacitances D. Inset: the readout error $N(D)$ for the pulse duration $t_p=12$ ns (circles) and $t_p=2$ ns (diamonds).

pulse width. A better situation can be achieved, if one substitutes the linear walls in the trapezoid by $\sin(2\pi t/t_p)$, see solid curves with triangles, diamonds and circles for amplitudes $A=0.034$, $A=0.032$, and $A=0.03$, respectively. In comparison with the purely sinusoidal pulse, $\sin(\pi t/t_p)$, one can gain about 15–20 % in the readout error, and, simultaneously, up to 50% in the readout speed, compare with the dashed curves, taken from Fig. 4 for $A=0.038$, $A=0.036$, $A=0.034$, and $A=0.032$, respectively. Further increase in the readout speed can be achieved, if one increases the duration of the flat part of the sine-trapezoid pulse from $t_p/8$ to $7t_p/8$ (see solid curve with crosses for $A=0.03$). Transferring to the limit of a rectangular pulse, one should stop somewhere, since further increase in the flat part of the pulse will give little increase in speed, but will obviously lead to an increase in the nonadiabatic error.[10]

Finally, let us briefly demonstrate how by adjusting the depth of a shallow potential well the readout error can be decreased further. If this well is deep enough, there are many levels inside, and while both Γ_0 and Γ_1 are small, their values are close to each other, which complicates discrimination between the states $|0\rangle$ and $|1\rangle$. If the well is too shallow, the tunneling from the state $|1\rangle$ may occur even without the driving pulse, and the discrimination between the states, again, will be pure. Therefore, there must be some optimal depth of the well, leading to the minimal readout error. The depth of the well can be adjusted either by variation in the constant magnetic field component a_0, or by variation in the capacitance C of the Josephson junction. The latter case is illustrated in Fig. 6, where the readout error N is presented for different amplitudes and values of the inverse capacitance $D=2e^2/\hbar C\times 10^{-9}$ in such a way that the minimum of N remains in approximately the same region of pulse duration $t_p\approx 12$ ns. Fixing t_p, from the main part of Fig. 6 one can extract the values of N and plot them as a function of D, as it is done in the inset of Fig. 6, the curve with circles. The readout error $N(D)$ demonstrates a pronounced minimum, and by choosing the optimal value of $D\approx 1.4$ Hz one can decrease the value of N down to 0.0355, leading to the fidelity $F=0.965$ (note that $N=0.0523$ for $D=0.6933$ Hz, $A=0.033$). The same procedure can be performed for $t_p=2$ ns (see the curve with diamonds in the inset): while for $D=0.6933$ Hz the fidelity was about 0.9, at the minimum, which is shifted to $D\approx 1.8$ Hz, $F\approx 0.94$. The location of the minimum of $N(D)$ corresponds to some value between three or two energy levels inside the shallow potential well. We note that a rather fast readout of 2ns duration with an acceptably high fidelity of 94% is achieved with a simple sinusoidal pulse, whose generation does not require complex pulse shaping hardware.[14] For 2 ns sine-trapezoid pulse the fidelity can be increased to 0.947. Thus, at the optimal point for the different pulse shapes we get nearly the same value of the error, while for longer pulses, Figs. 4 and 5, making the pulse top more flat leads to significant decrease in the error. This is again in close qualitative agreement with the results for the classical systems, where at the optimal point the sinusoidal pulses resulted in nearly the same level of switching error as the rectangular ones.[12] We, therefore, believe that the proper engineering of the qubit, together with the optimal adjustment of the readout pulse duration will finally lead to creation of high fidelity and high-speed readout qubits.

In conclusion, we have performed computer simulations of the process of pulsed readout from a two state quantum system, describing the phase qubit, within the model of one-dimensional time-dependent Schroedinger equation. It is demonstrated that by choosing the optimal pulse duration the readout error can be minimized. Further decrease in the readout error can be achieved by variation in the depth of a shallow potential well.

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